NOTE

ON A

CHANGE OF THE AXES

OF THE

TERRESTRIAL SPHEROID

IN RELATION TO THE

TRIANGULATION

OF THE

G. T. SURVEY OF INDIA.

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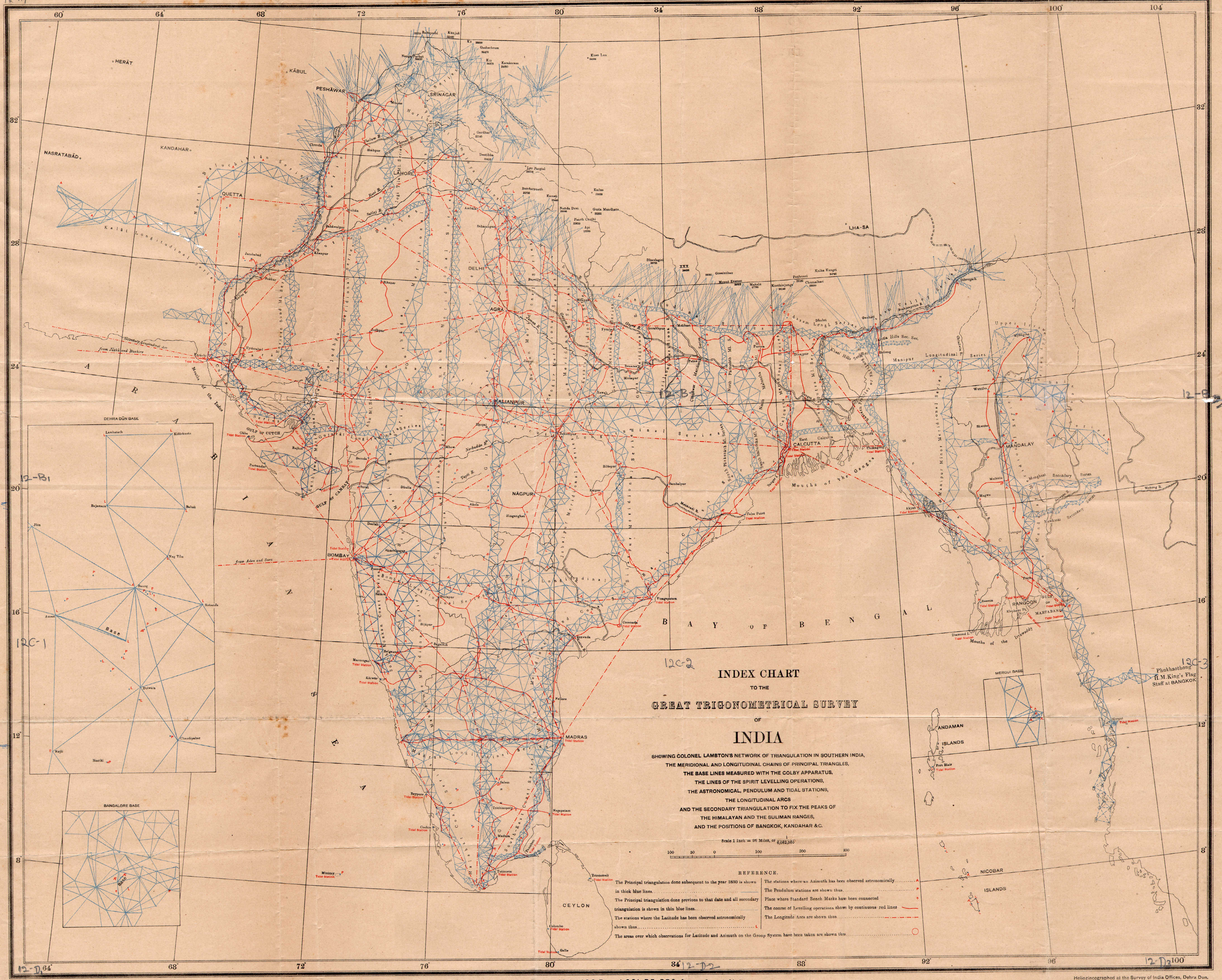

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NOTE ON A CHANGE OF THE AXES OF THE TERRESTRIAL SPHEROID IN RELATION TO THE TRIANGULATION OF THE GREAT TRIGONOMETRICAL SURVEY OF INDIA.

In the following note a method of finding the changes in coordinates (latitude, longitude and azimuth) of triangulated points, due to the adoption of recent values of the earth's axes is described.

The new axes are defined by

$$a = 6378200 \text{ metres (semi axis major)}$$
$$\epsilon = \frac{1}{298 \cdot 3} \text{ (compression or flattening).}$$

These values are given on p. 173 of "The Figure of the Earth and Isostasy, from Measurements in U.S.A." Washington 1909, and are said to be Dr. Helmert's latest values.

Heretofore the axes used in the G.T.S. are those due to Everest, known as "Everest's constants, first set". Their numerical values are—

$$a = 20{,}922{,}931 \cdot 80 \text{ feet}$$
$$\epsilon = \frac{1}{300 \cdot 8}.$$

All base lines of the G.T.S. have been expressed in terms of the Indian ten-foot standard, known as bar A. The base lines were not reduced to standard British feet but were given as some number of times $\frac{A}{10}$ feet. In making use of Everest's constants we have accordingly really been taking the semi-major axis as $20{,}922{,}931 \cdot 80 \frac{A}{10}$ British feet.

The value of A is given in G.T.S. Vol. I p. 28 as $3 \cdot 33331886\ Y$, Y being the British standard yard.

We accordingly have $\frac{A}{10} = 1 - \cdot 000004342$ from which we see that the semi major axis which has actually been used in India is $a = 20{,}922{,}840 \cdot 95$ feet.

2. Converting 6378200 metres into feet, by means of the relation

$$1 \text{ metre} = 39 \cdot 370113 \text{ inches}$$

deduced by Benoit, *see* "Raport du yard au métre" 1896 we get as our new value of a

$$20,\ 925,\ 871 \cdot 23,$$

and denoting by Δa the correction which has to be applied to the value used in the G.T.S., we have

$$\Delta a = + 3030 \cdot 26.$$

We also get

$$\Delta e^2 = \cdot 000,\ 055,\ 54 \quad \text{where } e \text{ is the eccentricity.}$$

3. While changing axes it is also proposed to slightly change the coordinates of Kalianpur, the origin of G.T.S.

Captain G.P. Lenox Conyngham, R.E. observed a group of latitudes and azimuths round Kalianpur. His results gave the mean value reduced to Kalianpur,

Lat. 24° 7′ 11″·57 Azimuth of Surantal 190° 27′ 6″·39.

The values heretofore adopted in the triangulation are

Lat. 24° 7′ 11″·26 Azimuth of Surantal 190° 27′ 5″·10.

We have to apply corrections to origin values of $+0''\cdot31$ in latitude and $+1''\cdot29$ in azimuth. As regards the old value of azimuth, a correction of $-1''\cdot1$ was applied to the observed value by General Walker in order to make azimuths observed at other parts of the triangulation agree with geodetic values. We are now annulling this by reverting to an observed value of azimuth.

4. We accordingly have to investigate equations giving the change in coordinates due to the changes exhibited in the following table.

	Old value	New value
Latitude of Kalianpur	24° 7′ 11″·26	24° 7′ 11″·57
Azimuth at Kalianpur of Surantal	190° 27′ 5″·10	190° 27′ 6″·39
Length of semi-axis major	20,922,840·95 feet	20,925,871·23 feet (=6378200 metres)
Compression	$\frac{1}{300\cdot8}$	$\frac{1}{298\cdot3}$

Any change in longitude of origin is of course immediately applicable to the whole of the triangulation, by simple addition (or subtraction).

5. The old triangulation was adjusted, that is to say its apparent errors were distributed, by a method following the method of least squares as closely as was thought to be practicable, in view of the great number of observed angles involved. Owing to the errors in the chosen values of the axes the equations which the errors were made to satisfy were not quite correct. In the first place the spherical excesses of the several triangles were computed on the slightly incorrect values of the axes: but on account of the smallness of these spherical excesses the change on this account is not appreciable to 0″·01, the accuracy to which they were computed. With the "circuit equations" the case is different. In following the series of triangulation which embrace much larger areas the spherical excess becomes much more appreciable, and its value on the new spheroid differs from the old value by about one second in an area of 75 square degrees.

This difference modifies the "circuit equations". It is fortunately much smaller than the errors generated in the triangulation.

The theoretically accurate course would be to readjust all the triangulation. This would be a very large piece of work and it is the object of present note to avoid this labour by putting forward an alternative method which will give the desired changes, in which any departure from strict theoretical accuracy will only give rise to errors of a considerably smaller order than those due to errors of observation. The method will also be applicable to any further changes that may be found to be desirable at some subsequent date.

6. The following is notation used:

a = semi major axis
b = ,, minor axis
ϵ = ellipticity or compression
e = eccentricity
ρ = radius of curvature to meridian
ν = normal terminated by minor axis

$$\beta^2 = \frac{\nu}{\rho}$$

ρ and ν are then the principal radii of curvature at a point, taking the figure to be a spheroid.

λ = latitude
L = longitude
A = azimuth measured from South by West
u = change in latitude due to change of origin and axes
v = ,, ,, longitude ,, ,,
w = ,, ,, azimuth ,, ,,
c = distance between points whose coordinates are λ, L and $\lambda + \Delta\lambda$, $L + \Delta L$.

As we shall only consider very small values of c it is unnecessary to specify whether this distance is measured along a normal plane section or a geodetic line.

Then we have, for sufficiently small values of c

$$\Delta\lambda = -\frac{c}{\rho}\cos A$$

$$\Delta L = -\frac{c}{\nu}\cdot\frac{\sin A}{\cos\lambda}$$

$$\Delta A = 180^\circ - \frac{c}{\nu}\sin A\tan\lambda.$$

The changes which take place in these quantities $\Delta\lambda$, ΔL, ΔA consequent on changes of A, λ, ρ, ν are given at once by differentiation:

$$\delta\Delta\lambda = \frac{c}{\rho}\cos A\frac{\delta\rho}{\rho} + \frac{c}{\rho}\sin A\,\delta A.$$

$$\delta\Delta L = \frac{c}{\nu}\cdot\frac{\sin A}{\cos\lambda}\cdot\frac{\delta\nu}{\nu} - \frac{c}{\nu}\cdot\frac{\cos A}{\cos\lambda}\cdot\delta A - \frac{c}{\nu}\cdot\frac{\sin A}{\cos^2\lambda}\cdot\sin\lambda\,\delta\lambda.$$

$$\delta\Delta A = \frac{c}{\nu}\sin A\tan\lambda\frac{\delta\nu}{\nu} - \frac{c}{\nu}\cos A\tan\lambda\,\delta A - \frac{c}{\nu}\sin A\sec^2\lambda.\ \delta\lambda$$

In these equations $\delta A = \Sigma\delta\,\Delta A$
$\delta\lambda = \Sigma\,\delta\Delta\lambda$

for δA is the azimuth change at the beginning of the element and is consequently the sum of the initial value at the beginning of the line and all the increments which occur in the line; and similarly for $\delta\lambda$. The quantities $\delta\lambda$, δL, δA which we shall denote by u, v, w are what we require to find. Putting in these letters we arrive at three simultaneous partial differential equations for finding u, v, w. To solve the equations it is necessary to assume some relation between λ, L and A defining the route along which we are to integrate. Perhaps a general solution might be obtained if a suitable relation was taken. But for the actual case it is sufficient to solve the equations for the two following cases:—

(1) when we proceed along a parallel of latitude,
(2) ,, ,, ,, meridian.

These cases correspond to $A = 90^\circ$ and $A = 0$. A means of dealing with the general case of an oblique curvilinear ray is given later.

Putting $A = 90^\circ$, we have $dw = d\Sigma\delta\Delta\lambda = \delta\Delta\lambda$ in limit etc.; also $\frac{c}{\nu\cos\lambda} = -dL$, when c is small, and equations may be written

$$\left.\begin{aligned} -\frac{du}{dL} &= \frac{\nu}{\rho}\cos\lambda.\ w \\ -\frac{dv}{dL} &= \frac{\delta\nu}{\nu} - \tan\lambda.\ w \\ -\frac{dw}{dL} &= \sin\lambda\,\frac{\delta\nu}{\nu}\cdot - \sec\lambda.\ w \end{aligned}\right\}$$

whence $\frac{d^2u}{dL^2} + \beta^2 u = \frac{\beta^2}{2}\cdot\sin 2\lambda.\frac{\delta\nu}{\nu}$, since $\beta^2 = \frac{\nu}{\rho}$.

7. The solution of this is

$$u = \tfrac{1}{2} \sin 2\,\lambda \frac{\delta\nu}{\nu} + P \cos(\beta L) + Q \sin(\beta L)$$

where P and Q are constants.

Whence $w = \dfrac{1}{\beta \cos\lambda} \cdot \left\{ P \sin(\beta L) - Q \cos(\beta L) \right\}$.

Determine P and Q by putting $L = 0$, and get

$$u_0 = \tfrac{1}{2} \sin 2\lambda \frac{\delta\nu}{\nu} + P$$

$$- Q = \beta w_0 \cos\lambda$$

where suffix zero denotes initial values at the beginning of the line.

Also $v - v_0 = -\dfrac{\delta\nu}{\nu} \cos^2\lambda\ L + \dfrac{\tan\lambda}{\beta} \left\{ P \sin(\beta L) + Q\,(1 - \cos\beta L) \right\}$

Expressing these equations in terms of seconds, — they are at present in radian units, — we write

$$R'' = \tfrac{1}{2} \sin 2\lambda \frac{\delta\nu}{\nu} \operatorname{cosec} 1''$$

$$P'' = u_0'' - R''$$

$$Q'' = -\beta\, w_0'' \cos\lambda$$

and the equations are

$$u'' = R'' + P'' \cos(\beta L) + Q'' \sin(\beta L)$$

$$v'' = v_0'' - \frac{\delta\nu}{\nu} \cos^2\lambda.\ L'' + \frac{\tan\lambda}{\beta} \left\{ P'' \sin(\beta L) + Q''\,(1 - \cos\beta L) \right\}$$

$$w'' = \frac{1}{\beta \cos\lambda} \left\{ P'' \sin(\beta L) - Q'' \cos(\beta L) \right\}.$$

In the above we require the value of $\dfrac{\delta\nu}{\nu}$. Now $\nu = \dfrac{a}{\sqrt{1 - e^2 \sin^2\lambda}},$ whence differentiating logarithmically with regard to a and e, expanding and putting in numerical values, we obtain the change due to the change of axes, *viz.*

$$\frac{\delta\nu}{\nu} = \cdot 000{,}144{,}83 + \cdot 000{,}027{,}77 \sin^2\lambda + \cdot 000{,}000{,}18 \sin^4\lambda + \;.\;.\;.\;.$$

8. Turning now to the second case when A = 0 we have at once $\dfrac{c}{\rho} = -\,d\,\lambda$

and the equations become

$$\frac{du}{d\lambda} = -\frac{\delta\rho}{\rho}$$

$$\frac{dv}{d\lambda} = \frac{\rho}{\nu} \sec\lambda.\ w$$

$$\frac{dw}{d\lambda} = \frac{\rho}{\nu} \tan\lambda.\ w$$

Now $\rho = \dfrac{a(1 - e^2)}{(1 - e^2 \sin^2\lambda)^{\frac{3}{2}}}$

whence differentiating with regard to a and e, expanding and putting in numerical values we obtain the change due to the change of axes, *viz.*

$$\frac{\delta\rho}{\rho} = \cdot 000{,}088{,}92 + \cdot 000{,}083{,}31 \sin^2\lambda + \cdot 000{,}000{,}55 \sin^4\lambda + \;.\;.\;.\;.$$

Then $- u\Big] = \displaystyle\int \frac{\delta\rho}{\rho} \cdot d\,\lambda$

$$= \cdot 000{,}130{,}78\,\lambda - \cdot 000{,}020{,}97 \sin^2\lambda + \cdot 000{,}000{,}02 \sin^4\lambda.$$

Expressing this in seconds and taking between limits we get

$$u'' - u_0'' = -\ \cdot 000{,}130{,}78\,(\lambda'' - \lambda_0'') + 4 \cdot 3254\,(\sin 2\lambda - \sin 2\lambda_0) - 0 \cdot 0035\,(\sin 4\lambda - \sin 4\lambda_0)$$

also $w'' = w_0'' \dfrac{\nu_0 \cos \lambda_0}{\nu \cos \lambda}$

$$v = \frac{\sqrt{1 - e^2}.\ \cos \lambda_0}{\sqrt{1 - e^2 \sin^2 \lambda_0}} \cdot \int \frac{dx}{(1 - x^2)^{\frac{3}{2}}\ \sqrt{1 - e^2 x^2}}$$

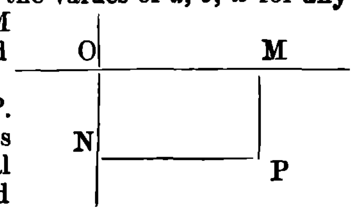
where $x = \sin \lambda$.

Expanding, integrating and changing to unit of second we get, neglecting small terms

$$v'' - v_0'' = w_0''. \frac{\nu_0 \cos \lambda_0}{a}. \sqrt{1 - e^2} \Big[\tan \lambda - \cdot 003{,}332{,}7\ \lambda'' \sin 1'' + \cdot 000{,}004{,}2 \sin 2\lambda \Big]_{\lambda_0}^{\lambda}$$

The value of $\log \sqrt{1 - e^2}$ is $\bar{1} \cdot 9985538$.

9. With the equations we have formed in §§7,8 we can now deduce the values of u, v, w for any point P. Starting from the origin O, we may compute along the parallel OM and find the values at M. Using these as initial values we can then proceed along the meridian MP and get values for P.



Or we may first proceed along meridian ON and then along parallel NP.

The values arrived at by the two routes are not identical. This is inevitable. The discrepancy in azimuth is the change in spherical excess of the given area from the old to the new spherical. We shall proceed to consider the discrepancies which occur. In the first place it was considered convenient to take as origin for this computation the point whose latitude and longitude were 24°, 78° on the old spheroid. The two values of the values of u, v, w, for this point differ by a small amount and in view of what follows the following mean value of w was taken:—

$$w = \frac{\dfrac{w_x}{x} + \dfrac{w_y}{y}}{\dfrac{1}{x} + \dfrac{1}{y}} = \frac{y w_x + x w_y}{x + y}$$

The suffix x indicates the value found by route ONP
and the suffix y „ „ „ „ „ „ OMP
and $x = \text{PN}$, $y = \text{PM}$, but x and y are always treated as positive. The values of u_x, u_y and v_x, v_y were practically identical.

Starting from this origin by means of our equations we obtain the values exhibited in the following three tables:—

TABLE I.

LATITUDE (u).

		58°	63°	68°	73°	78°	83°	88°	93°	98°
34°	u_x	−2·50858	−2·89432	−3·20019	−3·42385	*−3·56361*				
	u_y	*−2·07723*	*−2·65069*	*−3·09166*	*−3·39675*					
	Diff.	−0·43135	−0·24363	−0·10853	−0·02710	*0·00000*				
29°	u_x	−0·49755	−0·88329	−1·18916	−1·41282	*−1·55258*				
	u_y	*−0·27462*	*−0·75738*	*−1·13306*	*−1·39881*					
	Diff.	−0·22293	−0·12591	−0·05610	−0·01401	*0·00000*				
24°	u_x } u_y	*+1·40241*	*+1·01667*	*+0·71080*	*+0·48714*	*+0·34738*	*+0·29261*	*+0·32323*	*+0·43903*	*+0·63911*
	Diff.	*0·00000*	*0·00000*	*0·00000*	*0·00000*	*0·00000*	*0·00000*	*0·00000*	*0·00000*	*0·00000*
19°	u_x	+3·20512	+2·81938	+2·51351	+2·28985	*+2·15009*	+2·09532	+2·12594	+2·24174	+2·44182
	u_y	*+2·97072*	*+2·68699*	*+2·45452*	*+2·27510*		*+2·08045*	*+2·06672*	*+2·10900*	*+2·20697*
	Diff.	−0·23440	−0·13239	−0·05899	−0·01475	*0·00000*	+0·01487	+0·05922	+0·13274	+0·23485
14°	u_x	+4·92732	+4·54158	+4·23571	+4·01205	*+3·87229*	+3·81752	+3·84814	+3·96394	+4·16402
	u_y	*+4·44968*	*+4·27179*	*+4·11549*	*+3·98199*		*+3·78724*	*+3·72751*	*+3·69353*	*+3·68557*
	Diff.	−0·47764	−0·26979	−0·12022	−0·03006	0·00000	+0·03028	+0·12063	+0·27041	+0·47845

TABLE II.

LONGITUDE (v).

		58°	63°	68°	73°	78°	83°	88°	93°	98°
34°	v_x	+11·55396	+8·70715	+5·84003	+2·95734	+0·06397				
	v_y	+11·82474	+8·92142	+5·98821	+3·03302					
	Diff.	− 0·27078	−0·21427	−0·14818	−0·07568	0·00000				
29°	v_x	+10·97521	+8·23535	+5·47878	+2·70942	−0·06874				
	v_y	+11·03498	+8·28493	+5·51411	+2·72776					
	Diff.	− 0·05977	−0·04958	−0·03533	−0·01834	0·00000				
24°	v_x v_y	+10·45012	+7·80729	+5·15103	+2·48448	−0·18914	−2·86654	−5·54441	−8·21943	−10·88833
	Diff.	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000
19°	v_x	+ 9·96451	+7·41142	+4·84792	+2·27645	−0·30049	−2·88036	−5·46060	−8·03863	−10·61192
	v_y	+10·03969	+7·46470	+4·88195	+2·29303		−2·89693	−5·49462	−8·09189	−10·68705
	Diff.	+ 0·07518	+0·05328	+0·03403	+0·01658	0·00000	+0·01657	+0·03402	+0·05326	+ 0·07513
14°	v_x	+9·50738	+7·03876	+4·56259	+2·08063	−0·40531	−2·89337	−5·38170	−7·86843	−10·35171
	v_y	+9·78031	+7·23887	+4·69380	+2·14557		−2·95831	−5·51288	−8·06847	−10·62452
	Diff.	+0·27293	+0·20011	+0·13121	+0·06494	0·00000	+0·06494	+0·13118	+0·20004	+ 0·27281

TABLE III.

AZIMUTH (w).

		58°	63°	68°	73°	78°	83°	88°	93°	98°
34°	w_x	+5·83531	+4·75699	+3·64230	+2·49973	*+1·33803*				
	w_y	*+8·78306*	*+6·98775*	*+5·13898*	*+3·25090*					
	Diff.	−2·94775	−2·23076	−1·49668	−0·75117	0·00000				
29°	w_x	+5·53262	+4·51023	+3·45336	+2·37006	*+1·26862*				
	w_y	*+6·97539*	*+5·60208*	*+4·18593*	*+2·73775*					
	Diff.	−1·44277	−1·09185	−0·73257	−0·36769	0·00000				
24°	w_x, w_y	*+5·29810*	*+4·31905*	*+3·30698*	*+2·26960*	*+1·21485*	*+0·15080*	*−0·91440*	*−1·97260*	*−3·01574*
	Diff.	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000	0·00000
19°	w_x	+5·11995	+4·17383	+3·19579	+2·19329	*+1·17400*	+0·14573	−0·88365	−1·90627	−2·91433
	w_y	*+3·71867*	*+3·11339*	*+2·48428*	*+1·83619*		*+0·50282*	*−0·17217*	*−0·84588*	*−1·51311*
	Diff.	−1·40128	−1·06044	−0·71151	−0·35710	0·00000	−0·35709	−0·71148	−1·06039	−1·40122
14°	w_x	+4·99000	+4·06789	+3·11467	+2·13762	*+1·14420*	+0·14203	−0·86122	−1·85789	−2·84036
	w_y	*+2·20928*	*+1·96355*	*+1·70277*	*+1·42897*		*+0·85068*	*+0·55065*	*+0·24640*	*−0·05974*
	Diff.	−2·78072	−2·10434	−1·41190	−0·70865	0·00000	−0·70865	−1·41187	−2·10429	−2·78062

10. We shall denote the distances PN, PM (in any linear unit, not in angular units) by x and y.

So $$x = (L - L_0)\,\nu\cos\lambda.$$

$$y = \int_{\lambda_0}^{\lambda} \rho\, d\lambda.$$

By inspection of the numbers shown in tables *I*, *II*, *III*, the following equations are found to be approximately true:

$$u_x - u_y = Ax^2y$$
$$v_x - v_y = Bxy^2$$
$$w_x - w_y = Cxy$$

where A, B, C are quantities varying slightly with the latitude, but which may be treated as constants with their mean values over any area with which we shall need to deal.

The last equation simply expresses that the closing error in azimuth is equal to the change in spherical excess, and the two previous equations are consequences of this.

Now $u_x - u_y$ is what we will call the "closing error in latitude" in proceeding round the circuit OMPN; $v_x - v_y$ and $w_x - w_y$ being corresponding quantities for longitude and azimuth.

Then over any elementary area we have

$$\begin{aligned} dU &= d\ (u_x - u_y) = 2A\ x\ dxdy \\ dV &= d\ (v_x - v_y) = 2B\ y\ dxdy \\ dW &= d\ (w_x - w_y) = C\ dxdy \end{aligned}$$

If we integrate over any area we find the closing error of the circuit enclosing that area.

To find the values of u, v, w then which would be obtained by proceeding along any route we have merely to find the values of u_x, v_x, w_x (or u_y, v_y, w_y) and apply the closing error with the correct sign.

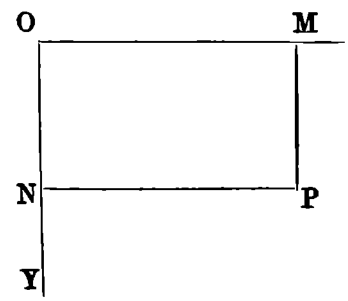
Integrating we have for *moderate areas*,

$$\begin{aligned} U &= 2A\ \bar{x}\ a \\ V &= 2B\ \bar{y}\ a \\ W &= Ca \end{aligned}$$

where a is the area of the circuit and $\bar{x}\,\bar{y}$ are the coordinates of its centre of gravity. We say for moderate areas because the coordinates x and y are curvilinear: but for the areas we shall require to apply the formulæ to, x and y may be treated as rectilinear coordinates.

11. By means of the above equations it is possible to find the result of the change of axes and origin as computed along any line of any curvature or along any route whatever, by computing first along a parallel and then along a meridian (or in the reverse order) and then applying the "closing errors" of the circuit formed by the line in question and the parallel and meridian.

This, then, would solve the problem as far as solitary lines were concerned. When we come to a network of lines the case is different, for we can find several values of the changes which occur at a point corresponding to the several possible routes by which the point can be reached. In view of the fact that most of the triangulation of India is along meridian or parallel (see triangulation chart at end), the following procedure is suggested :—



(1). Select central meridian and parallel for India (Burma will be dealt with separately).
The selected meridian is 78° and selected parallel 24°N.

(2). Assume the values of u, v, w found by the forumlæ on these lines, which we will call axes, to be correct.

(3). We have then to distribute the closing errors in PM and PN.

(4). If PM is a meridional series the computations fixing the *length* PM depend only in a small measure on the size of the earth's axes. The way in which these axes have come in is through the spherical excess. In nearly all triangulation in the Survey of India, the spherical excess is such a small quantity that the change of axes proposed will not appreciably affect it (to the $\frac{1}{100}$th of a second).
We have some ground then for assuming that the length PM is correct. In the same way the length PN may be regarded as correct. If then we take u_y and v_x for the changes in coordinates of P we should not be in error to the first order: and as the values of u, v are so small the second order quantities may surely be neglected.

(5). This process would hold for the corners of circuits formed by meridian and longitudinal series, though some modification would be more correct for oblique series. In the Indian triangulation meridional and longitudinal series are the rule. Oblique series occur practically only along the coast of the Bay of Bengal and along the first range of the Himalayas. (see index chart of triangulation at end).
As far as latitude and longitude are concerned we should not be committing much error in accepting the values of latitude and longitude, u_y and v_x.

(6). Now consider the azimuth change. This can be found from the change in position of two contiguous points. If we take two points originally on the same latitude whose changes are u_y and $u_y + \frac{\delta u_y}{\delta L}.\ dL$ the azimuth change on the line joining them is

$$\rho \frac{\delta u_y}{\delta L} dL . \frac{1}{\nu \cos \lambda .\ dL} = \frac{\rho}{\nu \cos \lambda} \cdot \frac{\delta u_y}{\delta L} = w_y$$

whereas the azimuth change deduced from two points originally on the same longitude is $\frac{\nu \cos \lambda}{\rho} . \frac{\delta v_x}{\delta \lambda} = w_x$.

Now we have seen that the azimuth closing errors is $C.\ x\, y$ where OM $= x$ ON $= y$ and C is a quantity which varies slightly with the latitude. Treating C as a constant and equal to its mean value over the area in consideration is permissible. This will be satisfied if the azimuth error is put into the lines PN and PM to amount proportional to their lengths.

We may therefore adopt $\dfrac{\frac{w_y}{y} + \frac{w_x}{x}}{\frac{1}{y} + \frac{1}{x}} = \dfrac{x w_y + y w_x}{x + y}$

as the best correction to the azimuth at P.

(7). The difference $w_y - w_x$ is not to be regarded as an error contained in the value of the angle M P N. Its effect is to alter the curvature of the lines P M and P N.

(8) In the case then of a gridiron system of meridional and longitudinal series at regular intervals all of equal weight, it seems that the best values we could assign to the changes are u_y, v_x, $\frac{x w_y + y w_x}{x + y}$. In this case the next step in correcting the triangulation would be to find the changes of intermediate points on the series as follows :—

u_y for change in latitude along a meridian
v_x for change in longitude along a parallel

and for the other coordinate and azimuth to simply interpolate between the terminal values.

(9) A difficulty arises when we come to consider the actual points of the triangulation series. For if we use the above formula for two adjacent points A and B, the difference of coordinates will not exactly give the correct change of azimuth. If we were to adopt the rule of computing the azimuth from the coordinates we should arrive at a different azimuth change on ray going east from that found on a ray going south. That is to say the actual change of $w_x - w_y$ would be forced into the single angle formed by these rays. To avoid this it appears better to take the azimuth change to be $\frac{x w_y + y w_x}{x + y}$ and the change in coordinates of *one point only* to be given by u_y, v_x and compute the coordinates of second adjacent point from this with the corrected azimuth (*i.e.* old azimuth $+ \frac{x w_y + y w_x}{x + y}$) and the old value of the distance c.

The computation alluded to in (9) may be performed with tables such as are given in the "Auxiliary Tables" prepared for the new values of the axes : or we may at once deduce the changes in the position of the second point B by differentiation of the equations for $\Delta\lambda$, ΔL and ΔA.

The equations are:—

$$\Delta\lambda = - \frac{c}{\rho} \cos A - \tfrac{1}{2} \frac{c^2}{\rho\nu} \sin^2 A \tan \lambda = \delta_1 \lambda + \delta_2 \lambda$$

$$\Delta L = - \frac{c}{\nu} . \frac{\sin A}{\cos \lambda} + \tfrac{1}{2} \frac{c^2}{\nu^2} . \frac{\sin 2 A \tan \lambda}{\cos \lambda} = \delta_1 L + \delta_2 L$$

$$\Delta A = 180^\circ - \frac{c}{\nu} .\ \sin A \tan \lambda + \tfrac{1}{4} . \frac{c^2}{\nu^2} . (1 + 2 \tan^2 \lambda) = 180^\circ + \delta_1 A + \delta_2 A$$

being those given in § 6 carried to an extra term in consideration of the larger value of c now contemplated.

Differentiating we have at once

$$\delta\Delta\lambda = \delta_1\lambda\left(-\frac{\delta\rho}{\rho} - \tan A.w\right) + \delta_2\lambda\left(-\frac{\delta\rho}{\rho} - \frac{\delta\nu}{\nu} + 2\cot A.w + \frac{2}{\sin 2\lambda}u\right)$$

$$\delta\Delta L = \delta_1 L\left(-\frac{\delta\nu}{\nu} + \cot A.w + \tan\lambda\, u\right) + \delta_2 L\left\{-\frac{2\delta\nu}{\nu} + 2\cot 2A.w + (\cot\lambda + 2\tan\lambda)\,u\right\}$$

$$\delta\Delta A = \delta_1 A\left(-\frac{\delta\nu}{\nu} + \cot A.w + \frac{2}{\sin 2\lambda}u\right) + \delta_2 A\left(-\frac{2\delta\nu}{\nu} + \frac{4\tan\lambda\sec^2\lambda}{1+2\tan^2\lambda}u + 2\cot 2A.w\right)$$

In above u, v, w are the values found for one end of the base A : the values for the other end B are then $u + \delta\Delta\lambda$, $v + \delta\Delta L$ $w + \delta\Delta A$.

A third method is to reach B by proceeding first along the parallel AC through A and then down the meridian CB through B, by means of the formulæ (or tables) already given: and then applying the closing error of the area ACB.

It appears, then, that the expressions u_y, v_x, $\frac{xw_y + yw_x}{x+y}$ may be taken to represent the changes in latitude, longitude and azimuth respectively of any point in India (excluding Burma) with the restriction that adjacent points must be treated differently, the changes for the second point being deduced by one of the three methods just explained. On this basis the results may be given in convenient tabular form. They will represent the changes with accuracy considerably greater than the accuracy with which the points can be considered to be fixed in space by triangulation.

12. These values are believed to be satisfactory for all the purposes for which they can be used. As far as map producing goes the discrepancies are negligible. For geodetic purposes we require to know the absolute corrections to latitudes, longitudes and azimuths of a base where a junction is to be made with another survey—such as the Russian survey, or the Burma survey. We can do this as described in § 11 for one end of the base and then compute the coordinates of the other end of the base from a knowledge of its length. In the case of Burma the triangulation has not yet been adjusted. It will no doubt be adjusted with the new values of the axes and made to fit on to the most eastern series of the North-East Quadrilateral, *viz.*, the Shillong Meridional Series, after this has been corrected for change of axes.

We also wish to know corrections to triangulated latitudes or azimuths at stations where these quantities have also been observed astronomically, so as to know the actual plumb-line deflections. As regards latitude we have uncertainty of perhaps $0''\cdot 1$ on account of axes change after leaving the central latitude by 10°, *i. e.* one part in 360,000 which is of the order of accuracy of our base-lines in India. The error generated in the triangulation must eventually be greater than this. The same argument holds as regards the azimuth, where the uncertainty of change due to change of axes, and due to error generated in triangulation are necessarily larger numbers when expressed in seconds of arc than occur in the latitude. The astronomic observations for azimuth are less precise, considered from point of view of plumb-line deflection, than the latitude observations. Apart from these considerations an error in plumb-line deflection in latitude of $0''\cdot 1$ is of little account. In India we have plumb-line deflections of over $40''$ and at least at present tenths of seconds are too minute to be taken account of in any discussion of deflections.

13. It seems then that the method sketched above is sufficiently precise for the geodetic uses to which the results can be put, and higher accuracy could not be applied with advantage to the results of triangulation. The method of § 11 is applicable to points which can be reached by either route (meridian or parallel) without the route departing out of the region of triangulation. Thus while it applies to all the triangulation in India which has been adjusted, it could not be fairly applied without modification to Burma, for this would imply the existence of triangulation across the Bay of Bengal. As the Burma triangulation remains to be adjusted, this does not matter and it will only be necessary to apply the method as far as the Shillong Meridional Series, which can be done very satisfactorily, the more so as our selected central latitude crosses this series.

14. The Survey of India has been asked by the Siamese Survey Department to furnish the best possible values of the coordinates of Bangkok. The way in which this has been done will serve as a good illustration of the method of using the closing error to determine the changes which occur along a route which is neither meridional nor longitudinal. As far as longitude 90° the route may be taken to follow the central parallel, latitude 24° (*see* triangulation chart at end). From these it proceeds along the Burma Coast Series down to latitude 13° 45′, and thence to Bangkok along latitude 13° 45′. In this case then we first compute along parallel 24° up to longitude 98°: we then proceed along meridian 98° down to latitude 13° 45′. The result at this point is found from tables I, II, III by interpolation to be

$$u_y = +\ 3\cdot 758$$
$$v_y = -\ 10\cdot 625$$
$$w_y = +\ 0\cdot 012$$

Now *treating longitude 98° as axis* from which x is measured, we evaluate the closing errors over the area between the Coast series and latitude 24° and meridian 98° and get

$$\Sigma\, Ax^2y = U = +\cdot 017$$
$$\Sigma\, Bxy^2 = V = -\cdot 026$$
$$\Sigma\, Cxy = W = +\cdot 426$$

Hence the changes at latitude 13° 45′, longitude 98°, as *determined by the route following the Burma Coast Series* are $u_y + U$, $v_y + V$, $w_y + W$.

One further correction remains. The Bangkok Series which emanates from this point is expressed in "preliminary terms"—it was computed from preliminary values of the base from which it emanates. Later values of this base, found after the Coast Series had been made to emanate from the adjusted value of its base, require the following changes to be applied to the beginning of the Bangkok Series, *viz.*

in	latitude	...	$-1''\cdot 80$
	longitude	...	$-0''\cdot 17$
	azimuth	...	$+5''\cdot 50$

Combining these we arrive at the changes to be made at latitude 13° 45°, longitude 98°.

	Latitude	Longitude	Azimuth
By parallel and meridian route	3·758	−10·625	+0·012
Correction to bring into terms of Burma Coast Series route ...	+0·017	− 0·026	+0·426
Correction to "preliminary terms"	−1·80	− 0·17	+5·50
Total ...	+1·98	−10·82	+5·94

With these initial values we compute along parallel 13° 45′ up to longitude 100° 33′ 3″·5 the old value of the longitude of Phukhao Thong Station* in Bangkok and we get the changes

$$u = +\ 1''\cdot 73$$
$$v = -12''\cdot 14$$
$$w = +\ 5''\cdot 7$$

To bring into Greenwich terms the further correction $-2'\ 27''\cdot 18$ is required to the longitude, the final corrections being

$+\ 1''\cdot 73$	in latitude
$-\ 2'\ 39''\cdot 32$	in longitude
$+\ 5''\cdot 7$	in azimuth

These values have been supplied to Royal Survey Department, Siam.

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* Phukhao Thong Station is the most easterly triangulated point shown on the triangulation chart.